Coreq Support for Section 1.1a

Topic 1: Properties of Exponents
(Video: Exponents)

An exponent is a shorthand notation for repeated factors. For example, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ can be written as 2^5. The expression 2^5 is called an **exponential expression**. The **base** of this expression is 2, and the **exponent** is 5.

base → 2^{5} → exponent $= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

If x is a real number and n is a positive integer, then x^n is the product of n factors of x.

$$x^n = \underbrace{x \cdot x \cdot x \cdot x \cdot x \cdot \ldots \cdot x}_{n \text{ factors of } x}$$

2 ^ 12

Product Rule for Exponents
If m and n are positive integers and a is a real number, then

$$a^m \cdot a^n = a^{m+n}.$$

$9x^2 \cdot 5x^6 = 45x^{2+6} = 45x^8$

Power Rule for Exponents
If m and n are positive integers and a is a real number, then

$$\left(a^m\right)^n = a^{mn}.$$

$(2^3)^4 = 2^{3 \cdot 4} = 2^{12} = 4096$

Power of a Product Rule
If n is a positive integer and a and b are real numbers, then

$$(ab)^n = a^n \cdot b^n.$$

$(4x^3)^3 = 4^3 (x^3)^3 = 64x^9$

Power of a Quotient Rule
If n is a positive integer, a and b are real numbers, and $b \neq 0$, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

$\left(\frac{4x}{5}\right)^3 = \frac{(4x)^3}{5^3} = \frac{4^3 x^3}{5^3} = \frac{64x^3}{125}$

Quotient Rule for Exponents
If m and n are positive integers, a is a real number, and $a \neq 0$, then

$$\frac{a^m}{a^n} = a^{\boxed{m-n}}.$$

$\frac{20x^{10}}{4x^7} = \frac{20}{4} x^{10-7} = 5x^3$

Zero Exponent Rule
If b is a real number such that $\boxed{b \neq 0}$, then $b^0 = 1$.

$2025^0 = 1$, $0^0 = ?$

$10^0 = 1$

$(-10)^0 = 1$

$-10^0 = -1$

$4x^0 =$

Topic 2: Simplifying Algebraic Expressions
(Video: Simplifying Algebraic Expressions)

An algebraic expression containing the sum or difference of like terms can be simplified by applying the distributive property. This is called **combining like terms**.

For example, consider the expression $3x+2x$. We can use the distributive property to rewrite the sum $3x+2x$ as a product.

$7x^2 - 3x^2 = 4x^2$

$$3x+2x=(3+2)x=5x$$

When simplifying an algebraic expression containing parentheses, we often use the distributive property twice, first to remove the parentheses and then to combine any like terms.

$$(7y+9)(y-6) = 7y^2 \underbrace{-42y+9y} - 54 = \underbrace{7y^2-33y-54}.$$

$$\overset{A}{2}x^2 + \overset{B}{7}x - \overset{C}{15} = \underbrace{2x^2+10x}\underbrace{-3x-15} = 2x(x+5)-3(x+5)$$

$$= (x+5)(2x-3)$$

$AC = 2\cdot(-15) = -30 \leftarrow$ product
$B = 7 \leftarrow$ sum

$\boxed{\begin{matrix}-3\\10\end{matrix}}$

Topic 3: Properties of Equality
(Video: Properties of Equality)

The **addition property of equality** guarantees that adding the same number to both sides of an equation creates an equation that has the same solution set as the original equation. Since subtraction is defined in terms of addition, this property also applies to subtracting the same number from both sides of an equation.

$$z - \cancel{24} = -36$$
$$\quad \cancel{+24} \quad +24$$
$$\boxed{z=-12}$$

Addition Property of Equality:
If a, b, and c are real numbers and $a=b$, then $a+c=b+c$.

The **multiplication property of equality** guarantees that multiplying both sides of an equation by the same nonzero number creates an equation that has the same solution set as the original equation. Since division is defined in terms of multiplication, this property also applies to dividing both sides of an equation by the same nonzero number.

Multiplication Property of Equality:
If a, b, and c are real numbers, $c \neq 0$, and $a=b$, then $ac=bc$.

$$\frac{x}{8} = 5$$

$$\cancel{8}\cdot\frac{x}{\cancel{8}} = 8\cdot 5 \Rightarrow \boxed{x=40}$$

Topic 4: Finding a Least Common Denominator

Given a set of fractions, the **least common denominator** is the smallest number that is divisible by each denominator.

$$\frac{13\cdot 20}{49\cdot 20} + \frac{3\cdot 70}{14\cdot 70} + \frac{17\cdot 49}{20\cdot 49} \to \frac{13}{7\cdot 7},\ \frac{3}{7\cdot 2},\ \frac{17}{2\cdot 2\cdot 5} \to LCD = 2^2\cdot 5^1\cdot 7^2 = 980$$

$$\hookrightarrow \frac{260}{980} + \frac{210}{980} + \frac{833}{980} = \frac{260+210+833}{980} = \frac{1303}{980}.$$

$$\left.\begin{array}{ccc|c} 49, & 14, & 20 & 7 \\ 7, & 2, & 20 & 7 \\ 1, & 2, & 20 & 2 \\ 1, & 1, & 10 & 10 \\ \hline 1, & 1, & 1 & \end{array}\right\} 980$$

Linear equation

NOT linear $\to \sqrt{p},\ \frac{2}{p+1},\ \underbrace{2p^1 + 10p^{-5.2}}$

$$\boxed{\frac{1}{10}(2p-1) > \frac{9}{4}p - \frac{p-9}{8}} \to LCD: \left.\begin{array}{ccc|c} 10, & 4, & 8 & 2 \\ 5, & 2, & 4 & 2 \\ 5 & 1 & 2 & 2 \\ 5 & 1 & 1 & 5 \\ \hline 1 & 1 & 1 & 40 \end{array}\right.$$

$$40\left(\frac{1}{10}\cdot(2p-1)\right) > 40\left(\frac{9}{4}p - \frac{p-9}{8}\right)$$

$$\frac{40}{10}(2p-1) > \frac{360}{4}p - \frac{40(p-9)}{8}$$

$$4(2p-1) > 90p - 5(p-9)$$

$$8p-4 > 90p - 5p + 45$$

$$\left.\begin{array}{l} 8\cancel{p-4} > 85p+45 \\ \quad \cancel{+4} \qquad\qquad +4 \end{array}\right\}$$

$$8p > \cancel{85p}+49$$

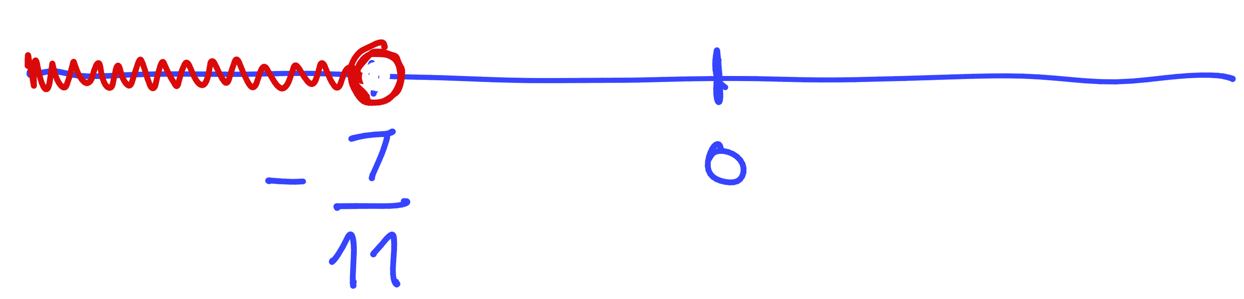
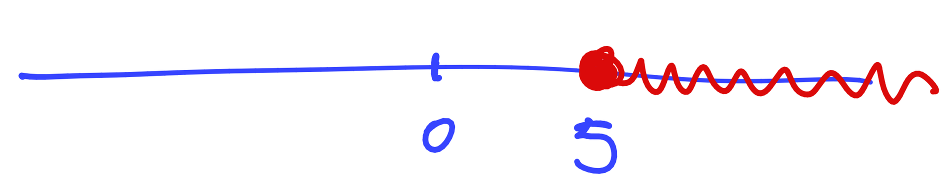
$$-85p \quad -\cancel{85p}$$

$$\frac{\cancel{-77p}}{\cancel{-77}} < \frac{49}{-77}$$

$$\boxed{p < -\frac{7}{11}}$$

$$\frac{1}{10}\left(2\left(\frac{-7}{11}\right)-1\right) = \frac{9}{4}\cdot\left(\frac{-7}{11}\right) - \frac{\left(\frac{-7}{11}\right)-9}{8}$$

exercise: verify.

ALGEBRAIC		INTERVAL		SET
$p < -\frac{7}{11}$	or		or	$\{p : p < -\frac{7}{11}\}$
$x \geq 5$	or		or	$\{x : x \geq 5\}$

$$\frac{7x}{x-6} + 1 = \frac{42}{x^2-36}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$\frac{7x}{x-6} + 1 = \frac{42}{(x-6)(x+6)} \rightarrow \text{LCD}: (x-6)(x+6)$$

$$(x-6)(x+6)\left(\frac{7x}{x-6} + 1\right) = \cancel{(x-6)(x+6)}\left(\frac{42}{\cancel{(x-6)(x+6)}}\right)$$

$$(x+6)(x-6) = x^2 - 6^2$$

$$(\cancel{x-6})(x+6)\frac{7x}{\cancel{x-6}} + (x-6)(x+6)\cdot 1 = 42$$

$$\Rightarrow (x+6)7x + (x-6)(x+6) = 42 \Rightarrow \underbrace{7x^2} + 42x + \underbrace{x^2} - 36 = 42$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -42 \quad -42$$

$$\frac{8x^2+42x-78}{2}=\frac{0}{2} \rightarrow 4x^2+21x-39=0$$

-156
21

$78\cdot 2$ $\quad 4\cdot 39 = 4\cdot(3\cdot 13) = 12\cdot 13 = 52\cdot 3$

$2\cdot 2\cdot 3\cdot 13$ $\quad 6\cdot 26$ $\quad 26-6$

$$4x^2+21x-39=0 \rightarrow x^2+21x-156=0$$

→ Is a prime expression

$$\frac{(3w-2)\cancel{(w-2)}\,w}{\cancel{w-2}} - \frac{\cancel{(3w-2)}(w-2)\,3w}{\cancel{3w-2}} = \frac{w-5\;\cancel{(3w-2)(w-2)}}{\cancel{3w^2-8w+4}}$$

$3w^2-8w+4$ → $(3w-2)(w-2)$

12
-8

-2
-6

$\underline{3w^2-2w}\;\underline{-6w+4}$

$w(3w-2)-2(3w-2)$

$(3w-2)(w-2)$

$(3w-2)w - (w-2)(3w) = w-5$

$\cancel{3w^2}-2w-\cancel{3w^2}+6w = w-5$

$4w = w-5$

$-w \quad -w$

$$\frac{3w}{3} = \frac{-5}{3}$$

$$\boxed{w=-\frac{5}{3}}$$

$$\frac{w}{w-2} - \frac{3w}{3w-2} = \frac{w-5}{(w-2)(3w-2)}$$

Restricted values: $w-2=0$ $\quad \boxed{w=2}$

$+2 \quad +2$

$3w-2=0 \rightarrow \frac{3w}{3}=\frac{2}{3} \rightarrow \boxed{w=\frac{2}{3}}$

$+2 \quad +2$

$$6x^2-x-2=0$$

$AC = 6\cdot(-2) = -12$ $\quad$ prod $\}$ $3\;\cancel{x}\;-4$

$B=-1$ $\quad$ sum

$1\cdot 12,\; 2\cdot 6,\; \boxed{3\cdot 4}$

$$\underbrace{6x^2-3x}\underbrace{+4x-2}=0$$

$$3x(2x-1)+2(2x-1)=0$$

$$\underset{a}{(2x-1)}\cdot\underset{b}{(3x+2)}=0 \rightarrow 2x-1=0 \text{ or } 3x+2=0$$

$+1 \quad +1 \qquad -2 \quad -2$

$\frac{2x}{2}=\frac{1}{2} \rightarrow \boxed{x=\frac{1}{2}}$ $\qquad \frac{3x}{3}=\frac{-2}{3} \rightarrow \boxed{x=-\frac{2}{3}}$